JHEP HYPER VERSION
Received: *November 4, 1998,* Accepted: *December 7, 1998*

Superstring scattering from D-branes bound states

Mohammad R. Garousi

Institute for Studies in Theoretical Physics and Mathematics IPM
P.O.Box 19395-5746, Tehran, Iran
Department of Physics, Birjand University,
Birjand, Iran
E-mail: mohammad@physics.ipm.ac.ir

Abstract: We derive fully covariant expressions for disk scattering amplitudes of any two massless closed strings in which mixed Neumann and Dirichlet world-sheet boundary conditions are included. From the two-point amplitudes, we derive the long range background fields and verify that they correspond to Dp-brane bound state. Also, from the scattering amplitudes, we calculate the linear coupling of closed string fields to D-brane world-volume and show that they are consistent with Born-Infeld and Chern-Simons actions in the presence of a background field.

Keywords: D-branes, String Duality.

JHEP12(1998)008

Contents

1. Introduction

Recent exciting progress in string theory has revealed many new connections between superstring theories which had previously been regarded as distinct theories [1]. Now it appears that all string theories are different phases of a single underlying theory, which also describes eleven dimensional supergravity [2]. Within these discussions, extended objects, other than just strings, play an important role. Hence these developments have generated a renewed interest in p-branes (*i.e.,* p-dimensional extended objects) and their interactions.

In perturbative type II superstring theories, there is a remarkably simple description of non-perturbative p-branes carrying Ramond–Ramond (R-R) charges [3]. The string background is taken to be simply flat empty space, however interactions of closed superstrings with these p-branes are described by world-sheets with boundaries fixed to a particular surface at the position of a p-brane. The latter is accomplished by imposing Dirichlet boundary conditions on the world-sheet fields [4, 5]. Hence these objects are referred to as Dirichlet p-branes (Dp-branes) or generically as simply D-branes. Within the type IIa theory, the Dp-branes can have $p = 0, 2, 4, 6$ or 8, while for the type IIb strings, p ranges over $-1, 1, 3, 5, 7,$ [3]. Scattering amplitudes describing the interactions of closed strings with D-branes was study extensively in [6, 7, 8, 9] where in [9] by studying scattering amplitude of massless closed string from D-branes it was verified that the long range background fields around these D-branes are those of extremally charged p-brane solutions of the low energy effective action.

Within the massless spectrum of open string states on a D-brane one finds a $U(1)$ gauge field [10]. If the D-brane carries a constant background gauge field, this background field induces new couplings on the D-brane to the R-R form potential, and the result may be regarded as a bound state of D-branes [11]. In scattering calculations or conformal field theory, this modifies the boundary conditions [10]. In type II superstring theory, interaction of closed superstrings with these D-brane bound states are described by the same world-sheet boundary conditions as for an ordinary D-brane except for world-volume directions that carry background fields. In these directions the Neumann boundary conditions are traded for mixed Neumann and Dirichlet boundary conditions. The interaction of massless superstrings with bound states of D-string and fundamental strings which is described by a D-string with its world-volume electric field turned on, was studied in [12]. The present paper provides calculations on scattering amplitude of any massless closed superstring from bound states of two D-branes which possess a difference in dimension of two using the mixed boundary condition. These bound states are described by Dp-branes with a world-volume magnetic field turned on.

The paper is organized as follows: In the following section we study modification of conformal field theory arising from mixed boundary conditions. From these study, we calculate specific D and M matrices that one needs in the subsequent sections. In section 3 we evaluate general amplitudes for scattering any two massless closed superstring states from a brane with arbitrary D and M matrices by using previously calculated open superstring amplitudes as was done in [9]. Our results include the scattering amplitudes with bosonic NS-NS and R-R states, and also fermionic NS-R and R-NS states. In section 4, we examine the massless closed string poles in the amplitudes for scattering from D-brane bound states. By comparing these terms to those in analogous field theory calculations, we are able to extract the long range background fields surrounding a Dp-brane bound state. Our calculations verify that these fields do correspond to those of extremally charged p- and $(p-2)$-brane bound state solutions of the low energy effective action [13]. Next we evaluate linear coupling of closed string fields to the D-brane bound state using the scattering amplitudes and show that as expected they are consistent with Born-Infeld and Chern-Simons actions in the presence of background gauge field. We conclude with a discussion of our results in Section 5.

2. Conformal field theory with background field

In perturbative superstring theories, to study scattering amplitude of some external string states in conformal field theory frame, one usually evaluate correlation function of their corresponding vertex operators with use of some standard conformal field theory propagators [14].[1] In trivial flat background one uses an appropriate linear σ-model to derive the propagators and define the vertex operators. While in nontrivial background fields, one must use nonlinear σ-model to do that. In nontrivial D-brane background the vertex operator remain unchanged whereas the standard propagators need some modification. Alternatively, one may use a doubling trick to convert the propagators to standard form and shift the modification to the vertex operators [9]. In [9] scattering amplitude of two massless closed string states in D-brane background was studied. There the D-brane was assumed to be flat. In this paper we would like to turning some of the D-brane background fields on and studying the scattering amplitude in the resulting nontrivial background. We consider D-branes that carry constant gauge and/or antisymmetric Kalb-Ramond field. The modifications arising from the appropriate σ-model appear in the following boundary conditions [10, 15]:[2]

$$\begin{aligned} \partial_y X^a - i\mathcal{F}^a{}_b \partial_x X^b &= 0 \text{ for } a, b = 0, 1, \ldots p\,, \\ X^i &= 0 \text{ for } i = p+1, \ldots 9\,, \end{aligned} \tag{2.1}$$

where $\mathcal{F}_{ab}$ are the constant background fields, and these equations are imposed at $y = 0$. In general one may choose to have non-zero background $\mathcal{F}_{ab}$ in all direction of D-brane but for simplicity we are interested in adding a constant background field in only two spatial direction of the D-brane world-volume *e.g.,* $\mathcal{F}_{12} = -\mathcal{F}_{21} = \mathcal{F}$. In this case the boundary condition (2.1) becomes

$$\begin{aligned} \partial_y X^c &= 0\,, \\ \partial_y X^1 - i\mathcal{F}\partial_x X^2 &= 0\,, \\ \partial_y X^2 + i\mathcal{F}\partial_x X^1 &= 0\,, \\ X^i &= 0\,, \end{aligned} \tag{2.2}$$

where $c = 0, 3, 4, \cdots p$. For special case of $\mathcal{F} = 0$, one has the Neumann boundary condition on all p dimensions which describe Dp-brane. The Dirichlet boundary condition on the remaining coordinates fixes the position of the Dp-brane at $X^i = 0$. While for large $\mathcal{F}$, the Neumann boundary condition are imposed only on $p-2$ dimensions which describe D$(p-2)$-brane. Here Dirichlet-like boundary conditions are imposed on two dimensions, *i.e.,* $\partial_x X^{1,2} = 0$, which does not fix position of the D$(p-2)$-brane in these dimensions. For arbitrary value of $\mathcal{F}$ one has both Dp-brane

[1] I am grateful for collaborations with Robert Myers in the results of this section.

[2] Our notation and conventions follow those established in [9]. So we are working on the upper-half plane with boundary at $y = 0$ which means ∂_y is normal derivative and ∂_x is tangent derivative.

and D($p-2$)-brane which the latter delocalized on the former, and the result may be considered as bound state of Dp- and D($p-2$)-branes.

Now we have to understand the modification of the conformal field theory propagators arising from these mixed boundary conditions. To this end consider the following general expression for propagator of $X^\mu(z,\bar{z})$ fields:

$$\begin{aligned}\langle X^\mu(z,\bar{z})\, X^\nu(w,\bar{w})\rangle &= -\eta^{\mu\nu}\log(z-w) - \eta^{\mu\nu}\log(\bar{z}-\bar{w}) - \\ &\quad - D^{\mu\nu}\log(z-\bar{w}) - D^{\nu\mu}\log(\bar{z}-w)\,,\end{aligned} \tag{2.3}$$

where $D^{\mu\nu}$ is a constant matrix that has to be fixed by imposing appropriate boundary condition. When both z and w are on the boundary of the world-sheet, *i.e.*, $z=\bar{z}\equiv x_1$ and $w=\bar{w}\equiv x_2$, the propagator (2.3) becomes

$$\langle X^\mu(x_1,x_1)\, X^\nu(x_2,x_2)\rangle = -2\left(\eta^{\mu\nu} + (D^S)^{\mu\nu}\right)\log(x_1-x_2) + i\pi(D^A)^{\mu\nu}\Theta(x_1-x_2)\,,$$

where D^S and D^A are the symmetric and antisymmetric parts of the D matrix, and $\Theta(x_1-x_2)=1(-1)$ if $x_1>x_2$ ($x_1<x_2$). Note that the last term in above equation which stems from the cut line of the Log function is zero for the systems that their $D^{\mu\nu}$ matrix is symmetric. Now the D matrix can be found for different boundary conditions. For the case that there is no boundary one finds $D^{\mu\nu}=0$, the standard propagators. In [9] the matrix $D^{\mu\nu}$ for flat D-brane was found to be $D_0^{\mu\nu}=V_0^{\mu\nu}+N^{\mu\nu}$ where $N^{\mu\nu}(V_0^{\mu\nu})$ projects vectors into subspace orthogonal(parallel) to D-brane. Using boundary conditions (2.2), one finds that the matrix $D^{\mu\nu}$ for the D-brane bound state is the same as $D_0^{\mu\nu}$ except in X^1 and X^2 directions, that is:

$$D^\mu{}_\nu = D_0{}^\mu{}_\lambda R^\lambda{}_\nu = R^\mu{}_\lambda D_0{}^\lambda{}_\nu\,,$$

where

$$R=\begin{pmatrix}\cos 2\theta & -\sin 2\theta\\ \sin 2\theta & \cos 2\theta\end{pmatrix}$$

is rotation matrix for angle 2θ. Here we defined $\cos 2\theta \equiv (1-\mathcal{F}^2)/(1+\mathcal{F}^2)$ and $\sin 2\theta \equiv 2\mathcal{F}/(1+\mathcal{F}^2)$. Note that this D matrix satisfies $D^\mu{}_\alpha D^{\alpha\nu}=\eta^{\mu\nu}$. Now writing $X^\mu(z,\bar{z})=X^\mu(z)+\tilde{X}^\mu(\bar{z})$, eq. (2.3) yields

$$\begin{aligned}\langle X^\mu(z)\, X^\nu(w)\rangle &= -\eta^{\mu\nu}\log(z-w)\,,\\ \langle \tilde{X}^\mu(\bar{z})\, \tilde{X}^\nu(\bar{w})\rangle &= -\eta^{\mu\nu}\log(\bar{z}-\bar{w})\,,\\ \langle X^\mu(z)\, \tilde{X}^\nu(\bar{w})\rangle &= -D^{\mu\nu}\log(z-\bar{w})\,.\end{aligned}$$

Similarly, propagator of ψ^μ fields, world-sheet super partner of X^μ, are modified as

$$\begin{aligned}\langle \psi^\mu(z)\, \psi^\nu(w)\rangle &= -\frac{\eta^{\mu\nu}}{z-w}\,,\\ \langle \tilde{\psi}^\mu(\bar{z})\, \tilde{\psi}^\nu(\bar{w})\rangle &= -\frac{\eta^{\mu\nu}}{\bar{z}-\bar{w}}\,,\\ \langle \psi^\mu(z)\, \tilde{\psi}^\nu(\bar{w})\rangle &= -\frac{D^{\mu\nu}}{z-\bar{w}}\,,\end{aligned}$$

and propagator of ghost fields remain unchanged and are

$$\begin{aligned}
\langle\phi(z)\,\phi(w)\rangle &= -\log(z-w)\,,\\
\langle\phi(z)\,\tilde{\phi}(\bar{w})\rangle &= -\log(z-\bar{w})\,,\\
\langle\tilde{\phi}(\bar{z})\,\tilde{\phi}(\bar{w})\rangle &= -\log(\bar{z}-\bar{w})\,.
\end{aligned}$$

These propagators can be transformed to the standard form by the doubling trick

$$\tilde{X}^\mu(\bar{z})\longrightarrow D^\mu{}_\nu X^\nu(\bar{z})\,,\qquad \tilde{\psi}^\mu(\bar{z})\longrightarrow D^\mu{}_\nu\psi^\nu(\bar{z})\,,\qquad \tilde{\phi}(\bar{z})\longrightarrow\phi(\bar{z})\,. \tag{2.4}$$

These replacements in effect extend the left-moving fields to the entire complex plane and shift modification arising from mixed boundary condition from propagators to vertex operators. Under replacements (2.4), propagator of the left-moving fields on the boundary become

$$\begin{aligned}
\langle X^\mu(x_1)\,X^\nu(x_2)\rangle &= -\eta^{\mu\nu}\log(x_1-x_2) - i\frac{\pi}{2}\mathcal{F}^{\mu\nu}\Theta(x_1-x_2)\,,\\
\langle\psi^\mu(x_1)\,\psi^\nu(x_2)\rangle &= -\eta^{\mu\nu}\log(x_1-x_2)\,,\\
\langle\phi(x_1)\,\phi(x_2)\rangle &= -\log(x_1-x_2)\,.
\end{aligned}$$

In scattering amplitude of boundary states, effect of the last term in the first line above is to add some phase factor to the scattering amplitude [12].

In order for studying scattering amplitude of all closed string states from D-brane with background field, one has to also know how to use the doubling trick for spin operator. Right-moving spin operator is replaced by

$$\tilde{S}_A(\bar{z})\longrightarrow M_A{}^B S_B(\bar{z})\,, \tag{2.5}$$

where the constant M matrix is defined by [9]

$$\gamma^\mu = (D^{-1})^\mu{}_\nu M^{-1}\gamma^\nu M\,. \tag{2.6}$$

Now using the decomposition of $D = D_0R$, one should be able to decompose $M = \Omega(R)M_0$ where M_0 is the standard M matrix which was derived in [9] (for D-brane with $\mathcal{F}=0$),

$$M_0^p = \begin{cases} \dfrac{\pm i}{(p+1)!}\,(\epsilon^v)_{\mu_0\cdots\mu_p}\,\gamma^{\mu_0}\cdots\gamma^{\mu_p} & \text{for } p+1 \text{ odd}\,,\\[2mm] \dfrac{\pm 1}{(p+1)!}\,(\epsilon^v)_{\mu_0\cdots\mu_p}\,\gamma^{\mu_0}\cdots\gamma^{\mu_p}\,\gamma_{11} & \text{for } p+1 \text{ even}\,. \end{cases} \tag{2.7}$$

where ϵ^v is the world-volume form of D-brane, and $\Omega(R)$ is the spinor rotation matrix for the angle 2θ. (Note that since D_0 and R commute, the order of Ω and M_0 in M should not be important.) To see this, using the properties of M_0 matrix, $\gamma^d M_0 = M_0\gamma^d$, one can write (2.6) as

$$\Omega^{-1}\gamma^d\Omega = D^d{}_e\gamma^e = R^d{}_e\gamma^e \tag{2.8}$$

where $d, e = 1, 2$. In component form it is

$$\Omega^{-1}\gamma^1\Omega = \gamma^1 \cos 2\theta - \gamma^2 \sin 2\theta\,,$$
$$\Omega^{-1}\gamma^2\Omega = \gamma^1 \sin 2\theta + \gamma^2 \cos 2\theta\,.$$

These equations can be solved for Ω matrix by using Baker-Hausdorff expansion and the result is

$$\Omega = exp[\theta\gamma^1\gamma^2] = \cos\theta + \gamma^1\gamma^2 \sin\theta\,.$$

Since $M_0\gamma^d\gamma^e = \gamma^d\gamma^e M_0$, one realizes that $M_0\Omega = \Omega M_0$ as anticipated above. Therefore M matrix for D-brane with constant background $\mathcal{F}$ field is

$$M^p = M_0^p \cos\theta - M_0^{p-2} \sin\theta\,,$$

This form for M matrix was also found in [16]. Some trigonometric identities can be used to show

$$\cos\theta = \frac{1}{\sqrt{1+\mathcal{F}^2}}\,, \qquad \sin\theta = \frac{\mathcal{F}}{\sqrt{1+\mathcal{F}^2}} \tag{2.9}$$

Now that D and M matrices have been found, one may use the replacement (2.4) and (2.5) to convert evaluation of scattering amplitude to standard conformal field theory calculation.

3. Two-point amplitudes

A systematic approach for studying scattering amplitude of all massless closed superstring states from D-brane was developed in [9] where it was shown that there is a direct relation between four-point amplitude of type I theory and two-point amplitude of type II theory in a D-brane background. Since the former amplitudes are well-known for all massless states, a simple transformation involving among other things D_0 and M_0 matrices gives the two-point amplitudes. The difference between scattering amplitude of massless closed string states from D-brane with $\mathcal{F} = 0$ and $\mathcal{F} \neq 0$ is in the specific form of D and M matrices, hence one may use the former results to evaluate scattering amplitude of the latter.

General form of amplitudes for scattering any two massless closed superstring states from a brane with unspecified D and M matrices was given in [9]. However, there we implicitly assumed that the D matrix being symmetric. We should release that assumption in order to include D-branes with constant background fields. Therefore, we redo those calculations with the most general form for D and M matrices. So we begin with the well-known four-particle open superstring amplitudes involving massless vectors as well as spinors which may be expressed in the form [17][3]

$$A(1,\,2,\,3,\,4) = -\frac{1}{2}g^2\frac{\Gamma(4k_1\!\cdot\!k_2)\Gamma(4k_1\!\cdot\!k_4)}{\Gamma(1+4k_1\!\cdot\!k_2+4k_1\!\cdot\!k_4)}K(1,\,2,\,3,\,4)\,. \tag{3.1}$$

[3]Here and in the subsequent amplitudes, we omit the Dirac delta-function which imposes momentum conservation.

The various kinematic factors are then given by

$$
\begin{aligned}
K(\zeta_1\,\zeta_2\,\zeta_3\,\zeta_4) &= -16k_2\cdot k_3k_2\cdot k_4\,\zeta_1\cdot\zeta_2\zeta_3\cdot\zeta_4 - 16k_1\cdot k_2\Big(\zeta_1\cdot k_4\zeta_3\cdot k_2\zeta_2\cdot\zeta_4+ \\
&\quad +\zeta_2\cdot k_3\zeta_4\cdot k_1\zeta_1\cdot\zeta_3 + \zeta_1\cdot k_3\zeta_4\cdot k_2\zeta_2\cdot\zeta_3 + \zeta_2\cdot k_4\zeta_3\cdot k_1\zeta_1\cdot\zeta_4\Big) + \\
&\quad +\Big\{1,2,3,4 \longrightarrow 1,3,2,4\Big\} + \Big\{1,2,3,4 \longrightarrow 1,4,3,2\Big\}\,, \qquad (3.2) \\
K(u_1,\,u_2,\,u_3,\,u_4) &= -2\,k_1\cdot k_2\,\bar u_2\gamma^\mu u_3\,\bar u_1\gamma_\mu u_4 + 2\,k_1\cdot k_4\,\bar u_1\gamma^\mu u_2\,\bar u_4\gamma_\mu u_3\,, \qquad (3.3) \\
K(u_1,\,\zeta_2,\,\zeta_3,\,u_4) &= 2i\sqrt{2}\,k_1\cdot k_4\,\bar u_1\gamma\cdot\zeta_2\gamma\cdot(k_3+k_4)\gamma\cdot\zeta_3u_4 - 4i\sqrt{2}\,k_1\cdot k_2\;\times \\
&\quad \times\Big(\bar u_1\gamma\cdot\zeta_3u_4\,k_3\cdot\zeta_2 - \bar u_1\gamma\cdot\zeta_2u_4\,k_2\cdot\zeta_3 - \bar u_1\gamma\cdot k_3u_4\,\zeta_2\cdot\zeta_3\Big)\,, \qquad (3.4) \\
K(u_1,\,\zeta_2,\,u_3,\,\zeta_4) &= -2i\sqrt{2}\,k_1\cdot k_4\,\bar u_1\gamma\cdot\zeta_2\gamma\cdot(k_3+k_4)\gamma\cdot\zeta_4u_3 - \qquad (3.5) \\
&\quad -2i\sqrt{2}\,k_1\cdot k_2\,\bar u_1\gamma\cdot\zeta_4\gamma\cdot(k_2+k_3)\gamma\cdot\zeta_2u_3\,.
\end{aligned}
$$

In translating these results to the two-point amplitudes in D-brane bound state background, schematically ones associates (some cyclic permutation of) $(1,2,3,4)$ in the open string amplitude with $(1_L,2_L,2_R,1_R)$ in the closed string amplitude, where here the subscripts L and R denote the left- and right-moving components of the closed string states. An appropriate transformation of momenta and polarization tensors from open to closed superstring states gives scattering amplitude of closed superstring states from the D-brane bound states. The D-brane scattering amplitudes then take a universal form

$$
A(1,\,2) = -i\,\frac{\kappa\,T_p\sqrt{1+\mathcal{F}^2}}{2}\,\frac{\Gamma(-t/2)\Gamma(2q^2)}{\Gamma(1-t/2+2q^2)}K(1,\,2)\,. \qquad (3.6)
$$

Here we normalized the amplitude by replacing $g^2 \longrightarrow i\kappa^{(2-\chi)}T_{p,p-2}$ where the Euler number $\chi = 1$ for disk and κ and $T_{p,p-2} = T_p\sqrt{1+\mathcal{F}^2}$ are the closed string and the D-brane coupling constants, respectively [12]. For later discussions, it is useful to divide the kinematic factor as

$$
K(1,\,2) = 2q^2\,a_1(1,\,2) + \frac{t}{2}\,a_2(1,\,2)\,. \qquad (3.7)
$$

Then $a_1(1,\,2)$ will be essentially the residue of the massless t-channel pole, which will become important for the analysis in sect. 4. Now, it simply remains to translate the kinematic factors (3.2)-(3.5) in the appropriate way.

3.1 NS-NS scattering amplitudes

We begin by calculating the amplitudes describing the scattering of two massless NS-NS states from a Dirichlet p-brane bound state (*i.e.,* the scattering of gravitons, dilatons or Kalb-Ramond (antisymmetric tensor) states). The amplitudes are calculated as two closed string vertex operator insertions on a disk with boundary conditions (2.2). The amplitude may be written as

$$
A \simeq \int d^2z_1\,d^2z_2\;\langle\,V_1(z_1,\bar z_1)\;V_2(z_2,\bar z_2)\,\rangle\,, \qquad (3.8)
$$

where the vertex operators are

$$
\begin{aligned}
V_1(z_1,\bar z_1) &= \varepsilon_{1\mu\nu}\,{:}V^\mu_{-1}(p_1,z_1){:}\;{:}\widetilde V^\nu_{-1}(p_1,\bar z_1){:}\,, \\
V_2(z_2,\bar z_2) &= \varepsilon_{2\mu\nu}\,{:}V^\mu_0(p_2,z_2){:}\;{:}\widetilde V^\nu_0(p_2,\bar z_2){:}\,.
\end{aligned}
\tag{3.9}
$$

The holomorphic components above are given by

$$
\begin{aligned}
V^\mu_{-1}(p_1,z_1) &= e^{-\phi(z_1)}\,\psi^\mu(z_1)\,e^{ip_1\cdot X(z_1)}\,, \\
V^\mu_0(p_2,z_2) &= \left(\partial X^\mu(z_2) + ip_2\cdot\psi(z_2)\psi^\mu(z_2)\right)\,e^{ip_2\cdot X(z_2)}\,.
\end{aligned}
\tag{3.10}
$$

The anti-holomorphic components take the same form as in eq. (3.10) but with the left-moving fields replaced by their right-moving counterparts – *i.e.,* $X(z)\to\tilde X(\bar z)$, $\psi(z)\to\tilde\psi(\bar z)$, and $\phi(z)\to\tilde\phi(\bar z)$. As usual, the momenta and polarization tensors satisfy

$$
p_i^2 = 0\,,\qquad p_i^\mu\,\varepsilon_{i\mu\nu} = 0 = \varepsilon_{i\mu\nu}\,p_i^\nu
$$

and the various physical states would be represented with

$$
\begin{aligned}
\text{graviton}: &\quad \varepsilon_{i\mu\nu} = \varepsilon_{i\nu\mu}, \qquad \varepsilon_{i\mu}{}^{\mu} = 0\,. \\
\text{dilaton}: &\quad \varepsilon_{i\mu\nu} = \frac{1}{\sqrt 8}\left(\eta_{\mu\nu} - p_{i\mu}\ell_{i\nu} - \ell_{i\mu}p_{i\nu}\right), \qquad \text{where } p_i\cdot\ell_i = 1\,. \\
\text{Kalb}-\text{Ramond}: &\quad \varepsilon_{i\mu\nu} = -\varepsilon_{i\nu\mu}\,.
\end{aligned}
\tag{3.11}
$$

In order for dealing with standard conformal field theory propagators, we use the doubling trick (2.4) which converts the vertex operators (3.9) to

$$
\begin{aligned}
V_1(z_1,\bar z_1) &= \varepsilon_{1\mu\lambda}D^\lambda{}_\nu\,{:}V^\mu_{-1}(p_1,z_1){:}\;{:}V^\nu_{-1}(D^T\!\cdot p_1,\bar z_1){:}\,, \\
V_2(z_2,\bar z_2) &= \varepsilon_{2\mu\lambda}D^\lambda{}_\nu\,{:}V^\mu_0(p_2,z_2){:}\;{:}V^\nu_0(D^T\!\cdot p_2,\bar z_2){:}\,,
\end{aligned}
$$

using only the expressions in eq. (3.10).

Now following [9], one finds the final result of scattering amplitude (3.6) by replacing

$$
\begin{aligned}
2k_1^\mu &\to p_1^\mu\,, \qquad 2k_4^\mu \to (D^T\!\cdot p_1)^\mu\,, \\
2k_2^\mu &\to p_2^\mu\,, \qquad 2k_3^\mu \to (D^T\!\cdot p_2)^\mu\,, \\
\zeta_{1\mu}&\otimes\zeta_{4\nu} \to \varepsilon_{1\mu\lambda}D^\lambda{}_\nu\,, \\
\zeta_{2\mu}&\otimes\zeta_{3\nu} \to \varepsilon_{2\mu\lambda}D^\lambda{}_\nu\,,
\end{aligned}
\tag{3.12}
$$

in eqs. (3.1) and (3.2). Hence, the final result may be written as

$$
A = -i\frac{\kappa\,T_p\sqrt{1+\mathcal F^2}}{2}\,\frac{\Gamma(-t/2)\Gamma(2q^2)}{\Gamma(1-t/2+2q^2)}\left(2q^2\,a_1 + \frac{t}{2}\,a_2\right), \tag{3.13}
$$

where $t = -2p_1 \cdot p_2$ is the momentum transfer to the brane, and $q^2 = \frac{1}{2} p_1 \cdot D \cdot p_1$ is the momentum flowing parallel to the world-volume of the brane. The kinematic factors above are:

$$
\begin{aligned}
a_1 = \; & \mathrm{Tr}(\varepsilon_1 \cdot D)\, p_1 \cdot \varepsilon_2 \cdot p_1 - p_1 \cdot \varepsilon_2 \cdot D \cdot \varepsilon_1 \cdot p_2 - p_1 \cdot \varepsilon_2 \cdot \varepsilon_1^T \cdot D^T \cdot p_1 - p_1 \cdot \varepsilon_2^T \cdot \varepsilon_1 \cdot D \cdot p_1 - \\
& - \frac{1}{2} (p_1 \cdot \varepsilon_2 \cdot \varepsilon_1^T \cdot p_2 + p_2 \cdot \varepsilon_1^T \cdot \varepsilon_2 \cdot p_1) + q^2\, \mathrm{Tr}(\varepsilon_1 \cdot \varepsilon_2^T) + \left\{ 1 \longleftrightarrow 2 \right\}, \qquad (3.14)
\end{aligned}
$$

$$
\begin{aligned}
a_2 = \; & \mathrm{Tr}(\varepsilon_1 \cdot D)\, (p_1 \cdot \varepsilon_2 \cdot D \cdot p_2 + p_2 \cdot D \cdot \varepsilon_2 \cdot p_1 + p_2 \cdot D \cdot \varepsilon_2 \cdot D \cdot p_2) + p_1 \cdot D \cdot \varepsilon_1 \cdot D \cdot \varepsilon_2 \cdot D \cdot p_2 - \\
& - \frac{1}{2} (p_2 \cdot D \cdot \varepsilon_2 \cdot \varepsilon_1^T \cdot D^T \cdot p_1 + p_1 \cdot D^T \cdot \varepsilon_1^T \cdot \varepsilon_2 \cdot D \cdot p_2) + q^2\, \mathrm{Tr}(\varepsilon_1 \cdot D \cdot \varepsilon_2 \cdot D) - \\
& - q^2\, \mathrm{Tr}(\varepsilon_1 \cdot \varepsilon_2^T) - \mathrm{Tr}(\varepsilon_1 \cdot D) \mathrm{Tr}(\varepsilon_2 \cdot D)\, (q^2 - t/4) + \left\{ 1 \longleftrightarrow 2 \right\}.
\end{aligned}
$$

Our notation is such that *e.g.*, $p_1 \cdot \varepsilon_2 \cdot \varepsilon_1^T \cdot D^T \cdot p_1 = p_1^\mu\, \varepsilon_{2\mu\nu}\, {\varepsilon_1}^{\lambda\nu}\, (D^T)_{\lambda\rho}\, p_1^\rho$. Conservation of momentum under replacement (3.12) becomes

$$
(p_1 + p_1 \cdot D + p_2 + p_2 \cdot D)^\mu = 0\,. \qquad (3.15)
$$

Now for the Dirichlet directions (*i.e.,* those orthogonal to the D-brane), the left hand-side automatically vanishes and so there is no momentum conservation in those directions. For the world-volume directions other than $\mu = 1, 2$, this equation yields as before that $(p_1 + p_2)^\mu = 0$. The equation for $\mu = 1, 2$ are more complicated, but also require that momentum is conserved in those directions. Thus just as in the case with $\mathcal{F} = 0$, momentum is conserved in the world-volume directions, *i.e.,* $(p_1 + p_2) \cdot V_0^\mu = 0$.

3.2 R-R boson amplitude

The next simple case is using eqs. (3.1) and (3.3) to calculate the amplitude describing two R-R states scattering from the D-brane bound state. The latter amplitude would be written as

$$
A \simeq \int d^2z_1\, d^2z_2\, \langle\, V_1(z_1, \bar{z}_1)\,,\; V_2(z_2, \bar{z}_2)\, \rangle
$$

where the vertex operators are

$$
V_i(z_i, \bar{z}_i) = (P_- \Gamma_{i(n)})^{AB} :V_{-1/2\,A}(p_i, z_i): \; :\widetilde{V}_{-1/2\,B}(p_i, \bar{z}_i):. \qquad (3.16)
$$

The holomorphic components above are given by

$$
V_{-1/2\,A}(p_i, z_i) = e^{-\phi(z_i)/2}\, S_A(z_i)\, e^{ip_i \cdot X(z_i)} \qquad (3.17)
$$

and the anti-holomorphic components have the same form, but with the left-moving fields replaced by their right-moving counterparts. As before we use eq. (2.4) to replace anti-holomorphic components $\tilde{X}^\mu$ and $\tilde{\phi}$ in $\widetilde{V}_{-1/2\,B}$ by their corresponding holomorphic components. Similarly, the right-moving spin field is replaced using (2.5) by its corresponding left-moving spin field. With replacement (2.5), only standard correlators of the spin fields [18, 14, 19] appear in the subsequent calculations. We have explicitly included the chiral projection operator $P_- = (1-\gamma_{11})/2$ in vertex operator (3.16), so that our calculations are always made with the full 32×32 Dirac matrices of ten dimensions. We have also defined

$$\Gamma_{i(n)} = \frac{a_n}{n!} F^i_{\mu_1\cdots\mu_n}\, \gamma^{\mu_1}\cdots\gamma^{\mu_n}\,, \tag{3.18}$$

where $a_n = i$ for the $n=2$ and 4 fields in the type IIa theory, while $a_n = 1$ for $n=1$, 3 and 5 in the type IIb theory. In eq. (3.18), $F^i_{\mu_1\cdots\mu_n}$ is the linearized n-form field strength with

$$\begin{aligned} F^i_{\mu_1\cdots\mu_n} &= i\, n\, p_{i[\mu_1}\varepsilon_{i\mu_2\cdots\mu_n]} \\ &= i\, p_{i\mu_1}\varepsilon_{i\mu_2\cdots\mu_n} \pm \text{ cyclic permutations}\,, \end{aligned}$$

where $p_i^2 = 0$ and $p_i^\mu\, \varepsilon_{i\mu\mu_3\cdots\mu_n} = 0$. Hence the appropriate substitutions for the open string amplitude (3.3) to derive the D-brane amplitude are

$$\begin{aligned} &2k_1^\mu \to p_1^\mu\,, \qquad 2k_4^\mu \to (D^T\!\cdot p_1)^\mu\,, \\ &2k_2^\mu \to p_2^\mu\,, \qquad 2k_3^\mu \to (D^T\!\cdot p_2)^\mu\,, \\ &u_{1A}\otimes u_{4B} \to (P_-\,\Gamma_{1(n)}M)_{AB}\,, \\ &u_{2A}\otimes u_{3B} \to (P_-\,\Gamma_{2(m)}M)_{AB}\,. \end{aligned} \tag{3.19}$$

The resulting kinematic factor is:

$$\begin{aligned} a_1^{R-R,R-R} &= -\frac{1}{2}\mathrm{Tr}\left(P_-\Gamma_{1(n)}M\gamma_\mu C^{-1}M^T\Gamma^T_{2(m)}C\gamma^\mu\right), \\ a_2^{R-R,R-R} &= \frac{1}{2}\mathrm{Tr}\left(P_-\Gamma_{1(n)}M\gamma_\mu\right)\mathrm{Tr}(P_-\Gamma_{2(m)}M\gamma^\mu\Big)\,, \end{aligned}$$

where the trace is over the 32×32 Dirac matrices of ten dimensions. Performing these traces, one would find the kinematic factor in terms of only momenta and polarization tensors.

3.3 NS-NS and R-R amplitude

The next case is calculating the amplitude describing one R-R and one NS-NS state scattering from a Dirichlet brane with non-zero $\mathcal{F}$ using eqs. (3.1) and (3.4). The appropriate substitutions to derive the Dirichlet amplitude are already derived for the previous amplitudes in eqs. (3.12) and (3.19)

$$\begin{aligned} &2k_1^\mu \to p_1^\mu\,, \quad 2k_4^\mu \to (D^T\!\cdot p_1)^\mu\,, \quad 2k_2^\mu \to p_2^\mu\,, \quad 2k_3^\mu \to (D^T\!\cdot p_2)^\mu\,, \\ &u_{1A}\otimes u_{4B} \to (P_-\,\Gamma_{1(n)}M)_{AB}\,, \qquad \zeta_{2\mu}\otimes\zeta_{3\nu} \to \varepsilon_{2\mu\lambda}D^\lambda{}_\nu\,. \end{aligned} \tag{3.20}$$

The resulting kinematic factor is then:

$$
\begin{aligned}
a_1^{R-R,NS-NS} &= \frac{i}{2\sqrt{2}}\mathrm{Tr}[P_-\Gamma_{1(n)}M\gamma^\nu\gamma\cdot(p_1+p_2)\gamma^\mu]\,(\varepsilon_2\cdot D)_{\mu\nu}\,,\\
a_2^{R-R,NS-NS} &= -\frac{2i}{\sqrt{2}}\Big[\mathrm{Tr}(P_-\Gamma_{1(n)}M\gamma\cdot D^T\cdot\varepsilon_2^T\cdot D^T\cdot p_2)-\mathrm{Tr}(P_-\Gamma_{1(n)}M\gamma\cdot\varepsilon_2\cdot D\cdot p_2)-\\
&\qquad -\mathrm{Tr}(P_-\Gamma_{1(n)}M\gamma\cdot D^T\cdot p_2)\,\mathrm{Tr}(\varepsilon_2\cdot D)\Big]\,. \qquad (3.21)
\end{aligned}
$$

This amplitude will be of particular interest in the following section for determining the background R-R fields in sect. 4.3.

3.4 Fermion amplitudes

Following [9], the kinematic factor for scattering of two fermions can also be evaluated and the result would be those in [9] with some of the D matrices replaced by D^T, that is

$$
\begin{aligned}
K^{R-NS,R-NS} &= i\sqrt{2}\,q^2\Big(\psi_2\cdot p_1\gamma\cdot D^T\cdot P_-\psi_1-\psi_2\cdot D\cdot\gamma\,p_2\cdot P_-\psi_1-\\
&\qquad -\psi_2{}^\mu\,\gamma\cdot D^T\cdot p_1\,P_-\psi_{1\mu}\Big)+\\
&\quad +i\frac{t}{4\sqrt{2}}\Big(\psi_2\cdot D\cdot\gamma\,\gamma\cdot(p_1+D^T\cdot p_1)\,\gamma\cdot D^T\cdot P_-\psi_1\Big)\,,\\
K^{NS-R,NS-R} &= i\sqrt{2}\,q^2\Big(\psi_1\cdot p_2\,M^{-1}\,\gamma\cdot M\,P_\pm\psi_1-\psi_2\,M^{-1}\cdot\gamma\,M\,P_\pm\,p_1\cdot\psi_2-\\
&\qquad -\psi_1{}^\mu\,M^{-1}\,\gamma\cdot p_2\,M\,P_\pm\psi_{2\mu}\Big)+\\
&\quad +i\frac{t}{4\sqrt{2}}\Big(\psi_1\,M^{-1}\cdot\gamma\,\gamma\cdot(p_2+D^T\cdot p_2)\,\gamma\cdot M\,P_\pm\psi_2\Big)\,,\\
K^{R-NS,NS-R} &= -i\frac{q^2}{\sqrt{2}}\Big(\psi_1\,P_+\cdot D_\nu\,\gamma^\mu\,\gamma\cdot(p_1+p_2)\,\gamma^\nu\,M\,\psi_{2\mu}\Big)-\\
&\quad -i\frac{t}{4\sqrt{2}}\Big(\psi_1\,P_+\cdot D\cdot\gamma\,\gamma\cdot(p_2+D^T\cdot p_2)\,\gamma\cdot M\,\psi_2\Big)\,,
\end{aligned}
$$

where ψ_1 and ψ_2 are spinor polarizations.

Up to here we never used properties of D and M matrices in evaluating these boson and fermion amplitudes, hence they satisfy for general D and M matrices. One gets various different results correspond to choosing an explicit D and M matrices and explicitly evaluating the amplitudes. For example, one may use these amplitudes to evaluate amplitudes for scattering any two massless string states from D-branes with background fields in more than one plane or the D-branes that appear in the Appendix. In the rest of this paper, we specify the D and M matrices to those that correspond to the D-branes with constant background field $\mathcal{F}$.

JHEP12(1998)008

4. Massless t-channel poles

Recall that in the scattering amplitudes, the momentum transfer to the D-brane bound state is $t = -(p_1 + p_2)^2$. Given the general form of the string amplitudes in eqs. (3.6-3.7), one can expand these amplitudes as an infinite sum of terms reflecting the infinite tower of closed string states that couple to the D-brane in the t-channel[4] (*i.e.,* terms with poles at $\alpha' t = \alpha' m^2 = 4n$ with $n = 0, 1, 2, \ldots$). For low momentum transfer, *i.e.,* $\alpha' t << 1$, the first term representing the exchange of massless string states dominates. In this case, eqs. (3.6 – 3.7) reduce to

$$A \simeq i\,\kappa\,T_p \sqrt{1+\mathcal{F}^2}\,\frac{a_1}{t}\,. \tag{4.1}$$

One can reproduce these long-range interactions with a calculation in the low energy effective field theory in which Dp-brane source terms are added to the field theory action.

4.1 NS-NS sources

The NS-NS sector is common to both type II superstring theories, and so the same low energy effective action describes the graviton, dilaton and Kalb-Ramond fields in both theories. The latter may be written as

$$I^{NS-NS} = \int d^{10}x\,\sqrt{-g}\,\left[\frac{1}{2\kappa^2}R - \frac{1}{2}(\nabla\phi)^2 - \frac{1}{6}\,H^2\,e^{-\sqrt{2}\kappa\phi}\right], \tag{4.2}$$

where $H_{\alpha\mu\nu} = \partial_\alpha B_{\mu\nu} + \partial_\nu B_{\alpha\mu} + \partial_\mu B_{\nu\alpha}$. Given this low energy effective action, one can calculate the different propagators, interactions, and subsequently scattering amplitudes for these three massless NS-NS particles. In doing so, one defines the graviton field by $g_{\mu\nu} = \eta_{\mu\nu} + 2\kappa\,h_{\mu\nu}$.

To consider the scattering of these particles from a Dp-brane bound state, we would supplement the low energy action with source terms for the brane as follows:

$$I^{NS-NS}_{source} = \int d^{10}x\,\left[S_B^{\mu\nu}\,B_{\mu\nu} + S_\phi\,\phi + S_h^{\mu\nu}\,h_{\mu\nu}\right]. \tag{4.3}$$

Note that at least to leading order, S_B, S_ϕ and S_h above will be δ-function sources which are only non-vanishing at $x^i = 0$ where x^i is position of D-brane. We begin by determining the dilaton source S_ϕ. To this end, we consider a scattering process in which an external dilaton is converted to a graviton. Examining the low energy action (4.2), one finds that the only relevant three-point interaction is one graviton coupling to two dilatons through the dilaton kinetic term. Thus, the only particle appearing in the t-channel is the dilaton, and hence this amplitude will uniquely determine S_ϕ. The field theory amplitude may be written

$$A'_{h\phi} = i\tilde{S}_\phi(k)\,\widetilde{G}_\phi(k^2)\,\widetilde{V}_{h\phi\phi}(\varepsilon_1, p_1, p_2)\,,$$

[4]We explicitly restore α' here. Otherwise our conventions set $\alpha' = 2$.

where $\tilde{S}_\phi(k)$ is the Fourier transform of the dilaton source,

$$\widetilde{G}_\phi(k^2) = -i/k^2 \tag{4.4}$$

is the dilaton's Feynman propagator, and

$$\widetilde{V}_{h\phi\phi} = -i\, 2\kappa\, p_2\!\cdot\!\varepsilon_1\!\cdot\! k$$

is the vertex factor for the graviton-dilaton-dilaton interaction. Here, ε_1 is the graviton polarization tensor, and $k^\mu = -(p_1+p_2)^\mu$ is the t-channel momentum. (We have not included in $A'_{h\phi}$ the Dirac delta-function which imposes momentum conservation in the directions parallel to the Dp-brane bound state.) The analogous string amplitude $A^{\phi h}$ is constructed from eq. (3.13) by inserting the appropriate external polarization tensors from eq. (3.11). To compare $A'_{h\phi}$ with the massless t-channel pole in $A^{\phi h}$, one needs to evaluate a_1 for graviton and dilaton. Inserting the gravitation and dilaton polarization tensors from eq. (3.11), we find

$$a_1^{\phi h} = \frac{2+\mathrm{Tr}(D)}{2\sqrt{2}}\, p_2\cdot\varepsilon_1\cdot p_2 + \cdots$$

where the dots represent the terms which are either proportional to k^2 and hence do not appear as simple pole or canceled against $a_2^{\phi h}$ term in the whole string amplitude (3.13). Now comparing $A'_{h\phi}$ with (4.1), one finds agreement by setting

$$\widetilde{S}_\phi(k) = -\frac{T_p\sqrt{1+\mathcal{F}^2}}{4\sqrt{2}}\,(2+\mathrm{Tr}(D)) = -\frac{T_p\sqrt{1+\mathcal{F}^2}}{2\sqrt{2}}(\cos(2\theta)+p-4)\,, \tag{4.5}$$

where we used that $\mathrm{Tr}(D) = 2\cos(2\theta)+2p-10$. Note that the source is a constant independent of k in agreement with the expectation that the position space source in eq. (4.3) is a delta-function in the transverse directions.

The graviton source S_h can be determined from either h-h or B-B scattering from the Dirichlet brane bound state. In the first, massless t-channel exchange is mediated by only a graviton, while in the second, both a graviton and dilaton propagate in the t-channel. The corresponding amplitude for B-B scattering is

$$A'_{BB} = i\widetilde{S}_h^{\mu\nu}(k)\,(\widetilde{G}_h)_{\mu\nu,\lambda\rho}(k^2)\,(\widetilde{V}_{hBB})^{\lambda\rho} + i\widetilde{S}_\phi(k)\,\widetilde{G}_\phi(k^2)\,\widetilde{V}_{\phi BB}\,, \tag{4.6}$$

where the graviton propagator (in Feynman-like gauge — see *e.g.,* [20]) and the three-point interactions are given by

$$\begin{aligned}
(\widetilde{G}_h)_{\mu\nu,\lambda\rho} &= -\frac{i}{2}\left(\eta_{\mu\lambda}\eta_{\nu\rho}+\eta_{\mu\rho}\eta_{\nu\lambda}-\frac{1}{4}\eta_{\mu\nu}\eta_{\lambda\rho}\right)\frac{1}{k^2}\,,\\
(\widetilde{V}_{hBB})^{\lambda\rho} &= -i\,2\kappa\left(\frac{1}{2}\left(p_1\!\cdot\! p_2\,\eta^{\lambda\rho}-p_1^\lambda p_2^\rho - p_1^\rho p_2^\lambda\right)\mathrm{Tr}(\varepsilon_1\!\cdot\!\varepsilon_2)-\right.\\
&\qquad - p_1\!\cdot\!\varepsilon_2\!\cdot\!\varepsilon_1\!\cdot\! p_2\,\eta^{\lambda\rho} + 2\,p_1^{(\lambda}\,\varepsilon_2{}^{\rho)}\!\cdot\!\varepsilon_1\!\cdot\! p_2 + 2\,p_2{}^{(\lambda}\,\varepsilon_1{}^{\rho)}\!\cdot\!\varepsilon_2\!\cdot\! p_1 +\\
&\qquad \left. +2p_1\!\cdot\!\varepsilon_2{}^{(\lambda}\,\varepsilon_1{}^{\rho)}\!\cdot\! p_2 - p_1\!\cdot\! p_2\,(\varepsilon_1^\lambda\!\cdot\!\varepsilon_2^\rho+\varepsilon_2^\lambda\!\cdot\!\varepsilon_1^\rho)\right),\\
\widetilde{V}_{\phi BB} &= -i\sqrt{2}\kappa\,(2p_1\!\cdot\!\varepsilon_2\!\cdot\!\varepsilon_1\!\cdot\! p_2 - p_1\!\cdot\! p_2\ \mathrm{Tr}(\varepsilon_1\!\cdot\!\varepsilon_2))\,,
\end{aligned} \tag{4.7}$$

where our notation is such that $\mathrm{Tr}(\varepsilon_1\cdot\varepsilon_2)=\varepsilon_1{}^{\mu\nu}\,\varepsilon_{2\nu\mu}$, $p_1\cdot\varepsilon_2^{\lambda}=p_{1\delta}\,\varepsilon_2^{\delta\lambda}$ and $\varepsilon_1{}^{\rho}\cdot p_2=\varepsilon_1^{\rho\delta}\,p_{2\delta}$. In extracting these three-point interactions from bulk action (4.2), we used on-shell properties of the Kalb-Ramond antisymmetric fields. Now again we must compare this result with the massless t-channel pole in the string amplitude (3.13) with an appropriate choice of polarization tensors. Unraveling $\widetilde{S}^{\mu\nu}$ from eq. (4.6) is simplified by noting that the only symmetric two-tensor available is

$$\widetilde{S}_h^{\mu\nu}(k^2)=a(k^2)\,V_0^{\mu\nu}+b(k^2)\,N^{\mu\nu}+c(k^2)V^{\mu\nu}+d(k^2)k^\mu k^\nu\,, \tag{4.8}$$

given the symmetries of the scattering process. Here, $V=V_0R^S$ and R^S is symmetric part of the rotation matrix $R(2\theta)$. Replacing (4.4), (4.5), (4.7) and (4.8) into eq. (4.6)

$$\begin{aligned}
A'_{BB}=&-\frac{i\kappa}{k^2}\Bigg[\Big(a+b+c-\frac{1}{2}\{a-b-c\}\mathrm{Tr}(D_0)-c\mathrm{Tr}(D^S)-\\
&\qquad-\frac{1}{2}T_p\sqrt{1+\mathcal{F}^2}\{2+\mathrm{Tr}(D^S)\}\Big)\,p_1\!\cdot\!\varepsilon_2\!\cdot\!\varepsilon_1\!\cdot\!p_2+\\
&\quad+4c\left(p_1\!\cdot\!D^S\!\cdot\!\varepsilon_2\!\cdot\!\varepsilon_1\!\cdot\!p_2+p_1\!\cdot\!\varepsilon_2\!\cdot\!D^S\!\cdot\!\varepsilon_1\!\cdot\!p_2+p_2\!\cdot\!D^S\!\cdot\!\varepsilon_1\!\cdot\!\varepsilon_2\!\cdot\!p_1\right)+\\
&\quad+2(a-b-c)\,(p_1\!\cdot\!D_0\!\cdot\!\varepsilon_2\!\cdot\!\varepsilon_1\!\cdot\!p_2+p_2\!\cdot\!D_0\!\cdot\!\varepsilon_1\!\cdot\!\varepsilon_2\!\cdot\!p_1+p_1\!\cdot\!\varepsilon_2\!\cdot\!D_0\!\cdot\!\varepsilon_1\!\cdot\!p_2)\times\\
&\quad\times\Big(-(a-b-c)p_1\!\cdot\!D_0\!\cdot\!p_2+2cp_1\!\cdot\!D^S\!\cdot\!p_1\Big)\,\mathrm{Tr}(\varepsilon_1\cdot\varepsilon_2)+8d\,p_1\!\cdot\!\varepsilon_2\!\cdot\!p_1\,p_2\!\cdot\!\varepsilon_1\!\cdot\!p_2\Bigg]\\
&-i\frac{\kappa}{4}\Bigg[\Big(-(a+b-3c)+\frac{1}{2}(a-b-c)\mathrm{Tr}(D_0)+c\mathrm{Tr}(D^S)+2d\,k^2\Big)\,\mathrm{Tr}(\varepsilon_1\!\cdot\!\varepsilon_2)-\\
&\quad-4(a-b-c)\mathrm{Tr}(D_0\!\cdot\!\varepsilon_1\!\cdot\!\varepsilon_2)-8c\mathrm{Tr}(D^S\!\cdot\!\varepsilon_1\!\cdot\!\varepsilon_2)-8d\,p_2\!\cdot\!\varepsilon_1\!\cdot\!\varepsilon_2\!\cdot\!p_1+\\
&\quad+\frac{T_p\sqrt{1+\mathcal{F}^2}}{2}\{2+\mathrm{Tr}(D^s)\}\mathrm{Tr}(\varepsilon_1\!\cdot\!\varepsilon_2)\Bigg]\,,
\end{aligned} \tag{4.9}$$

where D^S is symmetric part of the D matrix. The first bracket above is residue of the simple pole of D-brane amplitude and the second bracket contains some contact terms which are related to quadratic terms of source action (4.3).

Inserting the antisymmetric polarization tensors in (3.14), one may write it as

$$\begin{aligned}
a_1^{BB}\;=&\;-2\left(\frac{1}{2}p_1\!\cdot\!D^S\!\cdot\!p_1\,\mathrm{Tr}(\varepsilon_1\cdot\varepsilon_2)+p_1\!\cdot\!\varepsilon_2\!\cdot\!\varepsilon_1\!\cdot\!p_2+p_1\!\cdot\!D^S\!\cdot\!\varepsilon_2\!\cdot\!\varepsilon_1\!\cdot\!p_2+\right.\\
&\qquad\left.+p_2\!\cdot\!D^S\!\cdot\!\varepsilon_1\!\cdot\!\varepsilon_2\!\cdot\!p_1+p_1\!\cdot\!\varepsilon_2\!\cdot\!D^S\!\cdot\!\varepsilon_1\!\cdot\!p_2\right)\;.
\end{aligned}$$

Now comparing (4.9) with the massless t-channel pole (4.1) fixes a, b, c and d to be constants with $a=c=-\frac{1}{2}T_p\sqrt{1+\mathcal{F}^2}$ and $b=d=0$ leaving

$$\begin{aligned}
\widetilde{S}_h^{\mu\nu}\;&=\;-\frac{1}{2}T_p\sqrt{1+\mathcal{F}^2}\,(V_0^{\mu\nu}+V^{\mu\nu})\\
&=\;-T_p\sqrt{1+\mathcal{F}^2}\mathcal{V}^{\mu\nu}\,,
\end{aligned} \tag{4.10}$$

where $\mathcal{V}^{\mu\nu} = \mathrm{diag}(-1, 1/(1+\mathcal{F}^2), 1/(1+\mathcal{F}^2), 1, \ldots, 1, 0, \ldots, 0)$ where zeros appear in the transvers directions. As a cross check, we use this $\widetilde{S}_h^{\mu\nu}$ to calculate graviton-graviton scattering as well. The corresponding amplitude for $h-h$ scattering is

$$A'_{hh} = i\widetilde{S}_h^{\mu\nu}(k)\,(\widetilde{G}_h)_{\mu\nu,\lambda\rho}(k^2)\,(\widetilde{V}_{hhh})^{\lambda\rho}\,, \tag{4.11}$$

where the three-point interaction is

$$\begin{aligned}(\widetilde{V}_{hhh})^{\lambda\rho} = -i\,2\kappa\,\Big(&\Big(\frac{3}{2}p_1\!\cdot\! p_2\,\eta^{\lambda\rho} + p_1^{(\lambda}\,p_2^{\rho)} - k^\lambda\,k^\rho\Big)\,\mathrm{Tr}(\varepsilon_1\!\cdot\!\varepsilon_2) - \\ &- p_1\!\cdot\!\varepsilon_2\!\cdot\!\varepsilon_1\!\cdot\! p_2\,\eta^{\lambda\rho} + 2\,p_2^{(\lambda}\,{\varepsilon_2}^{\rho)}\!\cdot\!\varepsilon_1\!\cdot\! p_2 + 2\,{p_1}^{(\lambda}\,{\varepsilon_1}^{\rho)}\!\cdot\!\varepsilon_2\!\cdot\! p_1 + \\ &+ 2p_1\!\cdot\!{\varepsilon_2}^{(\lambda}\,{\varepsilon_1}^{\rho)}\!\cdot\! p_2 - p_1\!\cdot\! p_2\,(\varepsilon_1^\lambda\!\cdot\!\varepsilon_2^\rho + \varepsilon_2^\lambda\!\cdot\!\varepsilon_1^\rho) - \\ &- p_1\!\cdot\!\varepsilon_2\!\cdot\! p_1\,\varepsilon_1^{\lambda\rho} - p_2\!\cdot\!\varepsilon_1\!\cdot\! p_2\,\varepsilon_2^{\lambda\rho}\Big)\,.\end{aligned}$$

With this three-point interaction, the scattering amplitude (4.11) becomes

$$\begin{aligned}A'_{hh} = \frac{i\kappa}{k^2}T_p\sqrt{1+\mathcal{F}^2}\Big[&2p_1\!\cdot\!\varepsilon_2\!\cdot\!\varepsilon_1\!\cdot\! p_2 + 2p_2\!\cdot\!\varepsilon_1\!\cdot\!\varepsilon_2\!\cdot\! D^S\!\cdot\! p_2 + 2p_1\!\cdot\!\varepsilon_2\!\cdot\!\varepsilon_1\!\cdot\! D^S\!\cdot\! p_1 + \\ &+ 2p_1\!\cdot\!\varepsilon_2\!\cdot\! D^S\!\cdot\!\varepsilon_1\!\cdot\! p_2 - p_1\!\cdot\! D^S\!\cdot\! p_1\mathrm{Tr}(\varepsilon_1\cdot\varepsilon_2) - \\ &- \mathrm{Tr}(\varepsilon_1\!\cdot\! D^S)\,p_1\!\cdot\!\varepsilon_2\!\cdot\! p_1 - \mathrm{Tr}(\varepsilon_2\!\cdot\! D^S)\,p_2\!\cdot\!\varepsilon_1\!\cdot\! p_2 - \\ &- k^2\mathrm{Tr}(\varepsilon_1\!\cdot\!\varepsilon_2) - k^2\mathrm{Tr}(D^S\!\cdot\!\varepsilon_1\!\cdot\!\varepsilon_2)\Big]\,.\end{aligned} \tag{4.12}$$

Here again the terms in the last line above are contact terms and the other terms are residue of simple pole. Now the insertion of the graviton polarization tensor into a_1 gives

$$\begin{aligned}a_1^{hh} = &-2p_1\!\cdot\!\varepsilon_2\!\cdot\!\varepsilon_1\!\cdot\! p_2 - 2p_2\!\cdot\!\varepsilon_1\!\cdot\!\varepsilon_2\!\cdot\! D^S\!\cdot\! p_2 - 2p_1\!\cdot\!\varepsilon_2\!\cdot\!\varepsilon_1\!\cdot\! D^S\!\cdot\! p_1\,, \\ &-2p_1\!\cdot\!\varepsilon_2\!\cdot\! D^S\!\cdot\!\varepsilon_1\!\cdot\! p_2 + p_1\!\cdot\! D^S\!\cdot\! p_1\mathrm{Tr}(\varepsilon_1\cdot\varepsilon_2)\,, \\ &+\mathrm{Tr}(\varepsilon_1\!\cdot\! D^S)\,p_1\!\cdot\!\varepsilon_2\!\cdot\! p_1 + \mathrm{Tr}(\varepsilon_2\!\cdot\! D^S)\,p_2\!\cdot\!\varepsilon_1\!\cdot\! p_2\,.\end{aligned}$$

Comparing (4.12) with (4.1), one finds precise agreement between field theory calculation and low energy string scattering amplitude. One can also calculate dilaton-dilaton scattering in which the t-channel interaction is mediated by a graviton and the result is consistent with the result in eq. (4.10). Note however, this amplitude alone would not have enough structure to completely fix all of the unknown functions appearing in eq. (4.8).

To determine the antisymmetric tensor source, one can consider B-ϕ or B-h scattering. The scattering amplitude for B-h is

$$A'_{Bh} = i\widetilde{S}_B^{\mu\nu}\,(\widetilde{G}_B)_{\mu\nu,\lambda\rho}\,(\widetilde{V}_{BBh})^{\lambda\rho}\,, \tag{4.13}$$

where

$$\begin{aligned}(\widetilde{G}_B)_{\mu\nu,\lambda\rho} &= -\frac{i}{2}(\eta_{\mu\lambda}\eta_{\nu\rho} - \eta_{\mu\rho}\eta_{\nu\lambda})\frac{1}{k^2}\,, \\ (\widetilde{V}_{BBh})^{\lambda\rho} &= -i2\kappa\,\Big(p_2\!\cdot\!\varepsilon_1\!\cdot\! p_2\varepsilon_2^{\lambda\rho} + 2p_2\!\cdot\!\varepsilon_1^{[\lambda}\varepsilon_2^{\rho]}\!\cdot\! p_1 + 2p_2\!\cdot\!\varepsilon_1\!\cdot\!\varepsilon_2^{[\lambda}p_2^{\rho]} + \\ &\qquad + 2p_1\cdot p_2\varepsilon_2^{[\lambda}\cdot{\varepsilon_1}^{\rho]} + p_2^{[\lambda}\varepsilon_1^{\rho]}\!\cdot\!\varepsilon_2\!\cdot\! p_1 + p_1\!\cdot\!\varepsilon_2\!\cdot\!\varepsilon_1^{[\lambda}p_2^{\rho]}\Big)\,.\end{aligned}$$

JHEP12(1998)008

Here, ε_1 and ε_2 are graviton and the Kalb-Ramond polarization tensors, respectively. Since $(D^A)^{\mu\nu}$, antisymmetric part of the D-matrix, is the only available antisymmetric tensor, one may write the antisymmetric tensor source as

$$\tilde{S}_B^{\mu\nu} = a(k^2)(D^A)^{\mu\nu} .$$

Replacing this $B^{\mu\nu}$ source and above vertex and propagator in (4.13) one finds

$$\begin{aligned} A'_{Bh} = \frac{2i\kappa a}{k^2}\Big(& p_2\!\cdot\!\varepsilon_1\!\cdot\! p_2 \mathrm{Tr}(D^A\!\cdot\!\varepsilon_2) - 2p_2\!\cdot\!\varepsilon_1\!\cdot\! D^A\!\cdot\!\varepsilon_2\!\cdot\! p_1 - 2p_2\!\cdot\!\varepsilon_1\!\cdot\!\varepsilon_2\!\cdot\! D^A\!\cdot\! p_2 + \\ &+ 2p_1\,\varepsilon_2\!\cdot\!\varepsilon_1\!\cdot\! D^A\!\cdot\! p_2 + k^2\mathrm{Tr}(\varepsilon_2\!\cdot\!\varepsilon_1\!\cdot\! D^A)\Big) . \end{aligned} \tag{4.14}$$

Now a_1 for antisymmetric Kalb-Ramond and graviton is

$$\begin{aligned} a_1^{Bh} = & \mathrm{Tr}(\varepsilon_2\!\cdot\! D^A) p_2\!\cdot\!\varepsilon_1\!\cdot\! p_2 - 2p_1\!\cdot\!\varepsilon_2\!\cdot\! D^A\!\cdot\!\varepsilon_1\!\cdot\! p_2 \\ & + 2p_1\!\cdot\!\varepsilon_2\!\cdot\!\varepsilon_1\!\cdot\! D^A\!\cdot\! p_1 - 2p_2\!\cdot\!\varepsilon_1\!\cdot\!\varepsilon_2\!\cdot\! D^A\!\cdot\! p_2 . \end{aligned}$$

Replacing it in (4.1) and comparing with (4.14) fixes a to be constant with $a = -\frac{1}{2}T_p\sqrt{1+\mathcal{F}^2}$ leaving

$$\begin{aligned} \tilde{S}_B^{\mu\nu} &= -\frac{1}{2}T_p\sqrt{1+\mathcal{F}^2}(D^A)^{\mu\nu} \\ &= T_p\frac{\mathcal{F}^{\mu\nu}}{\sqrt{1+\mathcal{F}^2}} . \end{aligned} \tag{4.15}$$

From here one can see that if $\mathcal{F}=0$, source of $B^{\mu\nu}$ field vanishes and one would get the result of [9].

4.2 R-R source

We would also like to determine source of the Ramond-Ramond potential fields. The relevant terms in the low energy effective action are

$$I^{R-R} = \int d^{10}x\,\sqrt{-g}\,\sum_n\left(-\frac{8}{n!}\,F_{(n)}\!\cdot\! F_{(n)}\,e^{(5-n)\frac{\kappa}{\sqrt{2}}\phi}\right) . \tag{4.16}$$

For the type IIb superstring, the sum runs over $n = 1$, 3 and 5, while for the type IIa theory, the sum includes $n = 2$ and 4. An added complication is that $F_{(5)}$ should be a self-dual field strength for which no covariant action exists. The above non-self-dual action will yield the correct type IIb equations of motion when the self-duality constraint is imposed by hand [21] — *i.e.*, one makes the substitution $F_{(5)} \to F_{(5)} + *F_{(5)}$ in the equations of motion. One can verify that this action (4.16) reproduces the three-point string amplitudes on the sphere for two R-R vertex operators (3.17) scattering with a graviton or dilaton (3.9).

Now we supplement the above low energy action with the following source term

$$I_{\rm source}^{R-R} = \int d^{10}x[S_C^{\mu_1\cdots\mu_n}C_{\mu_1\cdots\mu_n}] , \tag{4.17}$$

where S_C is source of the R-R form potential $C_{(n)}$. This potential field is defined as $dC_{(n)} = F_{(n+1)}$ for "electric" and $dC_{(8-n)} = *F_{(n+1)}$ for "magnetic" components of R-R field strength $F_{(n+1)}$. Here $*F_{(n+1)}$ is also defined by

$$\begin{aligned}(F_{(n+1)})_{\mu_1\cdots\mu_{n+1}} &= (**F_{(n+1)})_{\mu_1\cdots\mu_{n+1}} \\ &= \frac{1}{(9-n)!}(*F_{(n+1)})^{\nu_1\cdots\nu_{9-n}}\epsilon^{10}_{\nu_1\cdots\nu_{9-n}\mu_1\cdots\mu_{n+1}} .\end{aligned}$$

To evaluate the R-R source S_C, we consider a scattering process in which an external R-R field is converted to graviton. The field theory amplitude may be written as

$$A'_{F_{(n+1)}h} = i(\widetilde{S}_C(k))^{\mu_1\cdots\mu_n}(\widetilde{G}_C(k^2))_{\mu_1\cdots\mu_n}{}^{\nu_1\cdots\nu_n}(\widetilde{V}_{CF_{(n+1)}h})_{\nu_1\cdots\nu_n} . \tag{4.18}$$

Here we wrote the external R-R potential field as its field strength. This will be convenient latter on for comparing with string amplitude. Now using action (4.16), the R-R propagator and the three-point interactions involving "electric" and "magnetic" fields are found to be

$$\begin{aligned}(\widetilde{G}_C(k^2))_{\mu_1\cdots\mu_n}{}^{\nu_1\cdots\nu_n} &= -\frac{in!}{16k^2}\eta^{[\nu_1}_{[\mu_1}\eta^{\nu_2}_{\mu_2}\cdots\eta^{\nu_n]}_{\mu_n]} , \\ (\widetilde{V}_{C_{(n)}F_{(n+1)}h})_{\nu_1\cdots\nu_n} &= -\frac{32\kappa}{n!}\Big[\varepsilon^{\lambda}_{2\mu}(F_{1(n+1)})_{\lambda\nu_1\cdots\nu_n}k^{\mu} - n\varepsilon^{\lambda}_{2\nu_1}(F_{1(n+1)})_{\lambda\nu_2\cdots\nu_n\mu}k^{\mu}\Big] , \\ (\widetilde{V}_{C_{(8-n)}(*F_{(n+1)})h})_{\nu_1\cdots\nu_{8-n}} &= -\frac{32\kappa}{(8-n)!}\Big[\varepsilon^{\lambda}_{2\mu}(*F_{1(n+1)})_{\lambda\nu_1\cdots\nu_{8-n}}k^{\mu} - \\ &\qquad - (8-n)\varepsilon^{\lambda}_{2\nu_1}(*F_{1(n+1)})_{\lambda\nu_2\cdots\nu_{8-n}\mu}k^{\mu}\Big] ,\end{aligned}$$

where ε_2 and $F_{1(n+1)}$ are the graviton polarization and the external R-R particle's (linearized) field strength, respectively. Now, since R-R potential is a total antisymmetric field, its source S_C must also be a total antisymmetric tensor. Using this and replacing above propagator and three-point interactions in (4.18), one obtains

$$A'_{F_{(n+1)}h} = -\frac{2\kappa(n+1)}{k^2}\varepsilon_2^{\lambda\mu}(F_{1(n+1)})_{\lambda}{}^{\nu_1\cdots\nu_n}k_{[\nu_n}(\widetilde{S}_C)_{\mu\nu_1\cdots\nu_{n-1}]} ,$$

for external "electric" field and

$$A'_{*F_{(n+1)}h} = -\frac{2\kappa(9-n)}{k^2}\varepsilon_2^{\lambda\mu}(*F_{1(n+1)})_{\lambda}{}^{\nu_1\cdots\nu_{8-n}}k_{[\nu_{8-n}}(\widetilde{S}_C)_{\mu\nu_1\cdots\nu_{7-n}]} ,$$

for external "magnetic" R-R field strength. The massless t-channel pole for the string scattering amplitude A_{hF} comes from $a_1^{R-R,NS-NS}$ in eq. (3.21) which is

$$\begin{aligned}a_1^{R-R,h} &= -\frac{i}{2\sqrt{2}}\mathrm{Tr}(P_-\Gamma_{1(n)}M^p\gamma^{\nu}\gamma{\cdot}k\gamma^{\mu})(\varepsilon_2{\cdot}D)_{\mu\nu} \\ &= \frac{i}{2\sqrt{2}}\mathrm{Tr}(\Gamma_{1(n)}\gamma^{\mu}\gamma{\cdot}kM^pP_+\gamma^{\nu})\varepsilon_{2\mu\nu} ,\end{aligned} \tag{4.19}$$

where in the second line above eqs. (2.6) and (3.15) have been used. Using (3.18) and (2.7), one may write eq. (4.19) as

$$
\begin{aligned}
a_1^{R-R,h} = \; & \frac{i}{4\sqrt{2}}\frac{a_n b_p}{n!(p+1)!}\,\varepsilon_{2\mu\nu}F_{1\mu_1\mu_2\cdots\mu_n}\,(\epsilon^v)_{\nu_0\nu_1\cdots\nu_{p'}}\,k_\alpha\times \\
& \times\Big\{Tr\Big[\gamma^{\mu_1}\gamma^{\mu_2}\cdots\gamma^{\mu_n}\gamma^{\mu}\gamma^{\alpha}\gamma^{\nu_0}\gamma^{\nu_1}\cdots\gamma^{\nu_{p'}}(1+\gamma_{11})\gamma^{\nu}\Big]\times \\
& \quad\times(\delta_{p',p}\cos\theta-\delta_{p',p-2}\sin\theta)\Big\}\,,
\end{aligned}
\tag{4.20}
$$

where $b_p=\pm i$ for $p+1=$ odd and $b_p=\pm 1$ for $p+1=$ even. Now one can perform the traces above for different n and p. The result is

$$
a_1^{R-R,h} = \pm i\frac{8\sqrt{2}}{n!}\varepsilon_2^{\lambda\mu}(F_{1(n+1)})_\lambda{}^{\nu_1\cdots\nu_n}\times\begin{cases}(n+1)k_{[\nu_n}(\epsilon^v)_{\mu\nu_1\cdots\nu_{n-1}]}\times \\ \qquad\qquad\times(\delta_{p,n-1}\cos\theta+\delta_{p,n+1}\sin\theta) \\ k^\rho\,(\epsilon^n)_{\rho\mu\nu_1\cdots\nu_n}(\delta_{p,7-n}\cos\theta+\delta_{p,9-n}\sin\theta)\end{cases}
$$

where the $\pm$ sign is the same as that appearing in (2.7). Here $\epsilon^v(\epsilon^n)$ is volume form in subspace parallel (orthogonal) to the brane's world-volume. Using identity

$$
(\epsilon^n)_{\mu_{p+1}\cdots\mu_9} = -\frac{1}{(p+1)!}(\epsilon^v)^{\mu_0\cdots\mu_p}\epsilon^{10}_{\mu_0\cdots\mu_p\mu_{p+1}\cdots\mu_9}\,,
$$

one may write $a_1^{R-R,h}$ as

$$
\begin{aligned}
a_1^{R-R,h} = \; & \pm i\frac{8\sqrt{2}(n+1)}{n!}\,\varepsilon_2^{\lambda\mu}\,(F_{1(n+1)})_\lambda{}^{\nu_1\cdots\nu_n}\,k_{[\nu_n}(\epsilon^v)_{\mu\nu_1\cdots\nu_{n-1}]}\times \\
& \qquad\qquad\qquad\times(\delta_{p,n-1}\cos\theta+\delta_{p,n+1}\sin\theta)\pm \\
& \pm i\frac{8\sqrt{2}(9-n)}{(8-n)!}\,\varepsilon_2^{\lambda\mu}\,(*F_{1(n+1)})_\lambda{}^{\nu_1\cdots\nu_{8-n}}\,k_{[\nu_{8-n}}(\epsilon^v)_{\mu\nu_1\cdots\nu_{7-n}]}\times \\
& \qquad\qquad\qquad\times(\delta_{p,7-n}\cos\theta+\delta_{p,9-n}\sin\theta)\,.
\end{aligned}
$$

Now comparing (4.1) with $A'_{F_{(n+1)}h}$, one finds the R-R "electric" sources as

$$
(\widetilde{S}_C)^E_{\mu_1\cdots\mu_n} = -\frac{4}{n!}Q_p\sqrt{1+\mathcal{F}^2}(\delta_{p,n-1}\cos\theta+\delta_{p,n+1}\sin\theta)(\epsilon^v)_{\mu_1\cdots\mu_n}
\tag{4.21}
$$

and the "magnetic" sources as

$$
(\widetilde{S}_C)^M_{\mu_1\cdots\mu_{8-n}} = -\frac{4}{(8-n)!}Q_p\sqrt{1+\mathcal{F}^2}(\delta_{p,7-n}\cos\theta+\delta_{p,9-n}\sin\theta)(\epsilon^v)_{\mu_1\cdots\mu_{8-n}}\,,
\tag{4.22}
$$

where $Q_p=\pm\sqrt{2}T_p$. In the case $\mathcal{F}=0$ ($\theta=0$), eq. (4.21) indicates that brans with $p=-1$, 0, 1, 2 and 3 carry "electric" charge of $C_{(n)}$ with $n=0$, 1, 2, 3 and 4, respectively. While eq. (4.22) shows that Dp-branes with $p=3$, 4, 5, 6 and 7 carry "magnetic" charge of $C_{(n)}$ with $n=4$, 5, 6, 7 and 8, respectively. As expected the D3-brane simultaneously carries "electric" and "magnetic" charge of the self-dual $F_{(5)}$ form.

4.3 Long range fields

From the dilaton, graviton, antisymmetric Kalb-Ramond and Ramond-Ramond sources, it is a simple matter to calculate (the Fourier transform of) the corresponding long range fields around the Dp-branes. These fields are precisely the product of the source and the Feynman propagator in the transverse momentum space. Hence the long range dilaton field is

$$\begin{aligned}\tilde{\phi}(k^2) &= i\widetilde{S}_\phi\, \widetilde{G}_\phi(k^2) \\ &= -\frac{T_p\sqrt{1+\mathcal{F}^2}}{4\sqrt{2}}\,\frac{2+\mathrm{Tr}(D)}{k^2} \\ &= -\frac{T_p\sqrt{1+\mathcal{F}^2}}{2\sqrt{2}}\,\frac{\cos(2\theta)+p-4}{k^2}\,. \end{aligned} \tag{4.23}$$

Similarly the long range gravitational field becomes

$$\begin{aligned}\tilde{h}_{\mu\nu}(k^2) &= i\widetilde{S}_h^{\lambda\rho}\,(\widetilde{G}_h)_{\lambda\rho,\mu\nu} \\ &= -\frac{T_p\sqrt{1+\mathcal{F}^2}}{8k^2}\mathrm{diag}\left(-\alpha_p,\gamma_p,\gamma_p,\alpha_p,\cdots,\alpha_p,\beta_p,\cdots,\beta_p\right), \end{aligned} \tag{4.24}$$

where $\alpha_p = -p+9-2\cos^2\theta$, $\gamma_p = 6\cos^2\theta - p + 1$ and $\beta_p = -p+1-2\cos^2\theta$. The long range antisymmetric tensor field is

$$\begin{aligned}\widetilde{B}_{\mu\nu}(k^2) &= i\widetilde{S}_B^{\lambda\rho}(\widetilde{G}_B)_{\lambda\rho,\mu\nu} \\ &= -\frac{T_p}{2k^2}\sqrt{1+\mathcal{F}^2}(D^A)_{\mu\nu} \\ &= \frac{T_p}{2k^2}\sqrt{1+\mathcal{F}^2}\sin(2\theta)(\epsilon^2)_{\mu\nu}\,, \end{aligned} \tag{4.25}$$

where ϵ^2 is volume form in the (x^1,x^2)-plane. Finally, the long range R-R potential fields are

$$\begin{aligned}\widetilde{C}_{\mu_1\cdots\mu_n}(k^2) &= i(\widetilde{S}_C)^{\nu_1\cdots\nu_n}(\widetilde{G}_C)_{\nu_1\cdots\nu_n,\mu_1\cdots\mu_n} \\ &= \frac{n!}{16k^2}(\widetilde{S}_C)_{\mu_1\cdots\mu_n} \\ &= -\frac{Q_p}{4k^2}\sqrt{1+\mathcal{F}^2}\left[\cos\theta\delta_{p,n-1}+\sin\theta\delta_{p,n+1}\right](\epsilon^v)_{\mu_1\cdots\mu_n}\,. \end{aligned} \tag{4.26}$$

From here one can see that a given $C_{(n)}$ simultaneously couples to both D$(p+1)$- and D$(p-1)$-branes.

Now we would like to compare above long range fields with the low-energy background field solutions corresponding to $(p+1)$- and $(p-1)$-brane bound states [13]. For simplicity we consider only the 2-brane and 0-brane bound state solution. In [13] this solution was found, by rotating a delocalized 1-brane in (x^1,x^2)-plane and using

T-duality on the rotated brane, to be (in the Einsten-frame)

$$
\begin{aligned}
d\hat{s}^2 &= e^{-\hat{\phi}/2}\sqrt{H}\left\{\frac{-dt^2}{H}+\frac{(dx^1)^2+(dx^2)^2}{1+(H-1)\cos^2\theta}+\sum_{i=3}^{9}(dx^i)^2\right\}\,,\\
\hat{C}_{(3)} &= \pm\frac{(H-1)\cos\theta}{1+(H-1)\cos^2\theta}dt\wedge dx^1\wedge dx^2\,,\\
\hat{C}_{(1)} &= \pm\frac{H-1}{H}\sin\theta dt\,,\\
\hat{B} &= \frac{(H-1)\cos\theta\sin\theta}{1+(H-1)\cos^2\theta}dx^1\wedge dx^2\,,\\
e^{2\hat{\phi}} &= \frac{H^{\frac{3}{2}}}{1+(H-1)\cos^2\theta}\,,
\end{aligned}
\tag{4.27}
$$

where $H=1+\mu G(r/\ell)$ and $G(r/\ell)=\frac{1}{5}(\ell/r)^5$ is the Green's function in the transverse space. Here $r^2=\sum_{i=3}^{9}(x^i)^2$ and ℓ is an arbitrary length. Also μ is some dimensionless constant and we consider it to be small, *i.e.,* $\mu << 1$, so that the nontrivial part of above solution may be treated as a perturbation of flat empty space. The perturbative parts, *i.e.,* $\mu << 1$, or long range fields, *i.e.,* $r\longrightarrow\infty$, of above solution are

$$
\begin{aligned}
\hat{h}_{\mu\nu} &\equiv \hat{g}_{\mu\nu}-\eta_{\mu\nu}\,,\\
&\simeq -\frac{\mu G}{8}\mathrm{diag}(-\alpha_2,\gamma_2,\gamma_2,\beta_2,\beta_2,\cdots,\beta_2)\,,\\
(\hat{C}_{(3)})_{\mu_1\mu_2\mu_3} &\simeq \pm\mu G\cos\theta(\epsilon^v)_{\mu_1\mu_2\mu_3}\,,\\
(\hat{C}_{(1)})_{\mu} &\simeq \pm\mu G\sin\theta(\epsilon^v)_{\mu}\,,\\
\hat{B}_{\mu\nu} &\simeq \mu G\cos\theta\sin\theta(\epsilon^2)_{\mu\nu}\,,\\
\hat{\phi} &\simeq -\frac{\mu G}{4}(2\cos^2\theta-3)\,,
\end{aligned}
\tag{4.28}
$$

where $\alpha_2=7-2\cos^2\theta$, $\gamma_2=6\cos^2\theta-1$ and $\beta_2=-1-2\cos^2\theta$. To make a precise comparison of the long range fields (4.23–4.26), with above asymptotic fields, we must first take into account that the low energy string actions, (4.2) and (4.16), have different normalization from that used in [13] to derive the bound state solution (4.27). The latter action would have the same normalization as the former if one uses the following field redefinitions

$$
\begin{aligned}
\hat{h}_{\mu\nu} &\equiv 2\kappa h'_{\mu\nu}\,,\\
\hat{\phi} &\equiv \sqrt{2}\kappa\phi'\,,\\
\hat{B}_{\mu\nu} &\equiv -2\kappa B'_{\mu\nu}\,,\\
\hat{C}_{(n)} &\equiv 4\sqrt{2}\kappa C'_{(n)}\,.
\end{aligned}
\tag{4.29}
$$

We have to also make Fourier transform of the asymptotic fields (4.28) in order to compare them with the momentum space long range fields (4.23– 4.26). The essential transform is that for $G(r/\ell)$ which is

$$\tilde{G}(k^2) \;=\; \frac{\mathcal{A}_6 \ell^5}{k^2}\,,$$

where $\vec{k}$ is a wave-vector in the subspace orthogonal to the 2-brane, and $\mathcal{A}_6$ is the area of a unit 6-sphere. Now in terms of the primed fields introduced in eq. (4.29), the Fourier transform of the asymptotic fields (4.28) are

$$\begin{aligned}
\tilde{h}'_{\mu\nu} &\simeq -\frac{\mu\mathcal{A}_6\ell^5}{16\kappa}\frac{1}{k^2}\mathrm{diag}(-\alpha_2,\gamma_2,\gamma_2,\beta_2\cdots,\beta_2)\,,\\
\tilde{\phi}' &\simeq -\frac{\mu\mathcal{A}_6\ell^5}{4\sqrt{2}\kappa}\frac{1}{k^2}(2\cos^2\theta-3)\,,\\
\tilde{B}'_{\mu\nu} &\simeq -\frac{\mu\mathcal{A}_6\ell^5}{2\kappa}\frac{1}{k^2}\cos\theta\sin\theta(\epsilon^2)_{\mu\nu}\,,\\
(\tilde{C}'_{(3)})_{\mu_1\mu_2\mu_3} &\simeq \pm\frac{\mu\mathcal{A}_6\ell^5}{4\sqrt{2}\kappa}\frac{1}{k^2}\cos\theta(\epsilon^v)_{\mu_1\mu_2\mu_3}\,,\\
(\tilde{C}'_{(1)})_{\mu} &\simeq \pm\frac{\mu\mathcal{A}_6\ell^5}{4\sqrt{2}\kappa}\frac{1}{k^2}\sin\theta(\epsilon^v)_{\mu}\,.
\end{aligned}$$

Hence we have complete agreement between the D-brane long range fields (4.23–4.26) and those above if $2\kappa T_p\sqrt{1+\mathcal{F}^2} \;=\; \mu\mathcal{A}_6\ell^5$. In [13], by evaluating the long range fields, mass and R-R charge of the $(p+1)$- and $(p-1)$-brane bound state was found and shown that they satisfy extremal condition. Therefore, the long range fields (4.23–4.26) are fields around extremally charged D$(p+1)$- and D$(p-1)$-brane bound state indicating as expected that the mixed boundary condition is world-sheet realization of the D-brane bound states.

4.4 D-brane action

In this section we show that the source actions derived from scattering amplitude of sect. (3) are consistent with Born-Infeld and Chern-Simons actions. To this end consider first the Born-Infeld action [5]

$$S_{BI} \;=\; -T_p\int d^{p+1}x\, e^{-\Phi}\sqrt{-\det\left(G_{ab}+\tilde{\mathcal{F}}_{ab}\right)}\,,$$

where G_{ab} and Φ are the string-frame metric and dilaton. Here $\tilde{\mathcal{F}}_{ab}$ contains background field and its quantum fluctuation. In order to compare above action with our result, one should write the Born-Infeld action in terms of the Einstein-frame metric and dilaton with the normalizations of the low energy action (4.2). Hence replacing $G_{ab}=e^{\Phi/2}g_{ab}$ and $\Phi=\sqrt{2}\phi$, one finds

$$S_{BI} \;=\; -T_p\int d^{p+1}x\, e^{\frac{p-3}{2\sqrt{2}}\phi}\sqrt{-\det\left(g_{ab}+e^{-\frac{\phi}{\sqrt{2}}}\tilde{\mathcal{F}}_{ab}\right)}\,.$$

[5]We are not interested in this paper in coupling of open string scalar and gauge fields to the D-brane.

Now we expand this action around backgrounds η_{ab} and $\mathcal{F}_{ab}$ which appear in $g_{ab} = \eta_{ab} + 2h_{ab}$ and $\tilde{\mathcal{F}}_{ab} = \mathcal{F}_{ab} - 2B_{ab}$, and keep only terms that are linear in h_{ab}, ϕ and B_{ab}. Using the following expansion

$$\sqrt{\det(A+\delta A)} = \sqrt{\det(A)}\left[1 + \frac{1}{2}\mathrm{Tr}(A^{-1}\delta A) + \cdots\right]$$

and identities

$$\sqrt{-\det\left(\eta_{ab} + e^{-\frac{\phi}{\sqrt{2}}}\mathcal{F}^{ab}\right)} = \sqrt{1 + e^{-\frac{2\phi}{\sqrt{2}}}\mathcal{F}^2}$$

$$(\eta_{ab} + \mathcal{F}_{ab})^{-1} = \mathcal{V}_{ab} - \frac{\mathcal{F}_{ab}}{1+\mathcal{F}^2}$$

one finds

$$\begin{aligned} S_{BI} = -T_p\sqrt{1+\mathcal{F}^2}\int d^{p+1}x\Bigg[1 + \mathcal{V}^{ab}h_{ab} + \left(\frac{p-3}{2} - \frac{\mathcal{F}^2}{1+\mathcal{F}^2}\right)\frac{\phi}{\sqrt{2}} \\ + \frac{\mathcal{F}^{ab}B_{ba}}{1+\mathcal{F}^2} + \cdots\Bigg], \end{aligned} \tag{4.30}$$

where the dots represent the linear coupling of open string fields to D-brane which can not be evaluated in our calculations, as well as higher order couplings. Here the indices are raised and lowered by η^{ab} and η_{ab}, respectively.

The Fourier transform sources (4.5), (4.10), (4.15), (4.21) and (4.22) are constant, so they appear as $S = \widetilde{S}\delta(x^i)$ in position space. Now replacing the NS sources in (4.3), one finds

$$I^{NS-NS}_{source} = -T_p\sqrt{1+\mathcal{F}^2}\int d^{p+1}x\left[\mathcal{V}^{ab}h_{ab} + \frac{1}{\sqrt{2}}\left(\frac{p-3}{2} - \frac{\mathcal{F}^2}{1+\mathcal{F}^2}\right)\phi - \frac{\mathcal{F}^{ab}B_{ab}}{1+\mathcal{F}^2}\right].$$

As can be seen, this action is the linear part of the Born-Infeld action (4.30)

The Chern-Simons action is

$$\begin{aligned} S_{CS} &= -\mu_p\int e^{\tilde{\mathcal{F}}}\sum c_{(p+1)} \\ &= -\mu_p\int dx^{p+1}\Bigg[\frac{1}{(p+1)!}c^{\mu_0\cdots\mu_p} + \frac{1}{(p-1)!2!}c^{\mu_0\cdots\mu_{p-2}}\tilde{\mathcal{F}}^{\mu_{p-1}\mu_p} \\ &\qquad + \frac{1}{(p-3)!2!2!2!}c^{\mu_0\cdots\mu_{p-4}}\tilde{\mathcal{F}}^{\mu_{p-3}\mu_{p-2}}\tilde{\mathcal{F}}^{\mu_{p-1}\mu_p} + \cdots\Bigg](\epsilon^v)_{\mu_0\cdots\mu_p}, \end{aligned}$$

where $c_{(n)}$ is the R-R form potential that has different normalization from that used in bulk action (4.16). Our calculations are consistent if one sets $c_{(n)} = 4C_{(n)}$. For special case of $\tilde{\mathcal{F}}_{\mu\nu} = \mathcal{F}_{\mu\nu}$ which has only two components, *i.e.*, $\mathcal{F}_{12}$ and $\mathcal{F}_{21}$, above action simplifies to

$$S_{CS} = -4\mu_p\int d^{p+1}x\left[\frac{1}{(p+1)!}C^{\mu_0\cdots\mu_p} + \frac{1}{(p-1)!2!}C^{\mu_0\cdots\mu_{p-2}}\mathcal{F}^{\mu_{p-1}\mu_p}\right](\epsilon^v)_{\mu_0\cdots\mu_p}. \tag{4.31}$$

Now writing sources (4.21) and (4.22) in position space and replacing them in (4.17), one finds

$$\begin{aligned} I^{R-R}_{source} &= -4Q_p \int d^{p+1}x\sqrt{1+\mathcal{F}^2} \\ &\quad \times \left[\frac{1}{(p+1)!}C^{\mu_0\cdots\mu_p}\cos\theta + \frac{1}{(p-1)!2!}C^{\mu_0\cdots\mu_{p-2}}(\epsilon^2)^{\mu_{p-1}\mu_p}\sin\theta\right](\epsilon^v)_{\mu_0\cdots\mu_p} \\ &= -4Q_p \int d^{p+1}x\left[\frac{1}{(p+1)!}C^{\mu_0\cdots\mu_p} + \frac{1}{(p-1)!2!}C^{\mu_0\cdots\mu_{p-2}}\mathcal{F}^{\mu_{p-1}\mu_p}\right](\epsilon^v)_{\mu_0\cdots\mu_p} \end{aligned}$$

where we used (2.9) to replace $\cos\theta$ and $\sin\theta$ in terms of $\mathcal{F}$. This action is actually the Chern-Simons action (4.31) by setting $\mu_p = Q_p$.

5. Discussion

In this paper, we have presented detailed calculations of all two-point amplitudes describing massless closed type II superstrings scattering from a Dirichlet p-branes that carry a background magnetic field. Using these results we derived the long range fields around the branes and showed as expected they are field around extremally charged D(p)- and D($p-2$)-brane bound states for $2 \le p \le 7$. From the scattering amplitude we also calculated linear coupling of closed string fields to the D-brane bound states and found that they are consistent with Born-Infeld and Chern-Simons actions.

The low-energy background field solutions which correspond to D-brane bound states presented in [13] possess one half of the supersymmetries of type II superstring theories. This happened because those solutions were constructed by imposing T-duality map on a single D-brane solution of type II theories. Supersymmetry is preserved by T-duality, hence the bound state solutions retain the supersymmetric properties of the initial (single) D-brane. In conformal field theory frame, as discussed in [9], any scattering procces that is describable in terms of D and M matrices has the same supersymmetry properties as type I theory. So (single) D-branes as well as the D-brane bound states that studied in this paper posses one half of the type II supersymmetries.

Using the ADM mass formula, mass per unit p-volume of the Dp- and D($p-2$)-brane bound state is $M_{p,p-2} = T_p\sqrt{1+\mathcal{F}^2}/\kappa \equiv T_{p,p-2}/\kappa$. Hence the D-brane bound states has tension $T_{p,p-2} = T_p\sqrt{1+\mathcal{F}^2}$, the one that used in the scattering amplitude (3.6). Similarly, R-R charges per unit p-volume are $Q_p = \pm\sqrt{2}T_p$ which was used in eqs. (4.21) and (4.22), and $Q_{p-2} = \pm\sqrt{2}T_p\mathcal{F}$. Therefore, they satisfy the extremal condition $(Q_p^2 + Q_{p-2}^2) = 2T_{p,p-2}^2$ [13].

From the gamma function factors appearing in eq. (3.13), we see that the amplitudes contain two infinite series poles corresponding to on-shell propagation of closed and open strings in t- and q^2-channels, respectively. The t-channel poles

($i.e.,$ $\alpha' t = 4n$ with $n = 0, 1, 2, \ldots$) indicate that the mass of closed strings are $(m^{closed})^2 = 4n/\alpha'$ because $t = -(p_1+p_2)^\mu (p_1+p_2)^\nu \eta_{\mu\nu} = (m^{closed})^2$ where $(p_1+p_2)^\mu$ is momentum of the closed strings. The q^2- or s-channel poles ($i.e.,$ $\alpha' q^2 = -n$) indicate that the open string masses are $(m^{open})^2 = n/\alpha'$. This is a result of the relation $q^2 = p_1^a p_1^b \mathcal{V}_{ab} = -(m^{open})^2$ where $p_1^a = (p_1 \cdot V_0)^\mu$ is the open string momentum. Hence the string tension seems to be shifted in an anisotropic manner on the D-brane world volume. The effect is to induce a non-trivial metric on its world-volume, $i.e.,$ one which is not just η_{ab}. Hence one might say that the D-brane world-volume is no longer flat. Of course, the closed strings propagating in the bulk remain unaffected by this shift, and hence they only see flat spacetime.

While our analyses of the R-R sources in sect. 4.2 are restricted to $2 \leq p \leq 7$, the NS-NS sources in sect. 4.1 are also valid in the case of domain walls (for which $p-8$). In this case as a result of "non-flat" nature of the world-volume metric $t \neq 4q^2$ and the kinematic factor $a_1(1,2)$ is not proportional to t, in contrast to the kinematic properties of amplitudes of scattering from single D-brane [9]. Also, the scattering amplitudes in field theory side do have non-zero simple poles which are equal to simple poles of string amplitude. Hence, for special case of $\mathcal{F} = 0$, our results verify that the NS-NS long range fields around D8-brane correspond to those of extremally charged ($p = 8$)-brane solution of the low energy effective action, things which we were not able to show in [9].

The t-channel analyses of sect. 4 enabled us to find linear coupling of closed string fields to the D-brane. The quadratic and higher order couplings appear as contact interactions. In general, to find these contact terms, one should equate contact terms of string amplitude to contact terms of field theory (with bulk and source actions) in both t- and s-channels. The two point amplitudes presented in sect. 3 has enough structure to extract the contact terms corresponding to two closed string fields. However, evaluation of these contact terms in field theory side in s-channel needs an understanding of coupling of open string gauge and scalar fields to D-brane [22]. It would be also interesting to evaluate coupling of massless open string fields to the D-brane bound state and show that they are consistent with Born-Infeld and Chern-Simons actions [23].

Finally, although the coupling of closed string fields to the D-brane world-volume that we found in this paper can be extracted from abelian Born-Infeld and Chern-Simons actions, extending the boundary condition (2.1) to includes non-abelian gauge fields helps one to find non-abelian Born-infeld and Chern-Simons actions [24], things which are not fully understood yet [25].

Acknowledgments

I gratefully aknowledge contributions of Robert C. Myers to an early stage of this work, and for suggesting improvements to a previous draft of this paper. I would also like to acknowledge useful conversations with H. Arfaei and J.C. Breckenridge. This work was supported by University of Birjand and IPM.

A. Moving D-branes

In this appendix we would like to evaluate the D and M matrices for moving D(p)-brane and D(p)-brane with its electric field turned on. To this end, we begin with a D(p)-brane that move with constant velocity v in one of its transvers direction, say X^1. Then appropriate boundary condition for the open strings on this brane is

$$\begin{aligned} \partial_y(X^0 - vX^1) &= 0\,, \\ X^1 - vX^0 &= 0\,, \\ \partial_y X^a &= 0\,, \\ X^i &= 0\,, \end{aligned}$$

where $a = 2, 3, ..., p+1$ and $i = p+2, ..., 9$. Now following the steps in sect. 2, one finds

$$(D_v^p)^{\mu\nu} = (D_0^{p+1})^\mu{}_\alpha R_v^{\alpha\nu}\,,$$

where

$$R_v^{\alpha\nu} = -\begin{pmatrix} \cosh 2\theta_v & \sinh 2\theta_v \\ \sinh 2\theta_v & \cosh 2\theta_v \end{pmatrix}$$

and we defined $\cosh 2\theta_v \equiv (1+v^2)/(1-v^2)$ and $\sinh 2\theta_v \equiv 2v/(1-v^2)$. The M matrix is also found to be

$$M_v^p = i\gamma^{11}(\gamma^1 \cosh\theta_v - \gamma^0 \sinh\theta_v) M_0^{p+1}\,,$$

where $D_0^{p+1}(M_0^{p+1})$ is the appropriate $D(M)$ matrix for stationary D(p+1)-brane which includes X_1 as its world-volume.

To write the D and M matrices for D(p)-branes which carry constant electric field, we consider the electric field to be non-zero only in X^1 direction. Then appropriate boundary condition can be read from (2.1) to be

$$\begin{aligned} \partial_y X^0 + iE\partial_x X^1 &= 0\,, \\ \partial_y X^1 + iE\partial_x X^0 &= 0\,, \\ \partial_y X^b &= 0\,, \\ X^j &= 0\,, \end{aligned}$$

where $b = 2, 3, ..., p$ and $j = p+1, ..., 9$. From these boundary conditions, one finds the D matrix as

$$(D_E^p)^{\mu\nu} = (D_0^p)^\mu{}_\alpha R_E^{\alpha\nu}\,,$$

where

$$R_E^{\alpha\nu} = -\begin{pmatrix} \cosh 2\theta_E & -\sinh 2\theta_E \\ \sinh 2\theta_E & -\cosh 2\theta_E \end{pmatrix}$$

and we defined $\cosh 2\theta_E \equiv (1+E^2)/(1-E^2)$ and $\sinh 2\theta_E \equiv 2E/(1-E^2)$. The M matrix can be evaluated and the result is

$$M_E^p = (\cosh\theta_E + \gamma^0\gamma^1 \sinh\theta_E) M_0^p\,.$$

Note that in both cases the D matrix satisfy $D^\mu{}_\alpha D^{\alpha\nu} = \eta^{\mu\nu}$. A difference between these two D matrices is that the $(D_v^p)^{\mu\nu}$ is symmetric while the $(D_E^p)^{\mu\nu}$ is not. So in scattering amplitude of boundary states from D(p)-branes which carry electr ic field the antisymmetric part of $(D_E^p)^{\mu\nu}$ produces some phase factors [12].

References

[1] see for example:
J. Polchinski, *String duality: a colloquium,* *Rev. Mod. Phys.* **68** (1996) 1245 [hep-th/9607050];
C. Vafa, *Lectures on strings and dualities,* hep-th/9702201.

[2] see for example:
M.J. Duff, *M-Theory (the theory formerly known as strings),* *Int. J. Mod. Phys.* **A 11** (1996) 5623 [hep-th/9608117];
J.H. Schwarz, *Lectures on superstring and M-theory dualities,* *Nucl. Phys.* **55 B** *(Proc. Suppl.)* (1997) 1 [hep-th/9607201].

[3] J. Polchinski, *Phys. Rev.* **D 75** (1995) 4724 [hep-th/9510017].

[4] J. Dai, R.G. Leigh and J. Polchinski, *Mod. Phys. Lett.* **A 4** (1989) 2073;
J. Polchinski, *Phys. Rev.* **D 50** (1994) 6041 [hep-th/9407031].

[5] J. Polchinski, S. Chaudhuri and C.V. Johnson, *Notes on D-branes,* hep-th/9602052;
J. Polchinski, *TASI lectures on D-Branes,* hep-th/9611050.

[6] I.R. Klebanov and L. Thorlacius, *Phys. Lett.* **B 371** (1996) 51 [hep-th/9510200].

[7] S.S. Gubser, A. Hashimoto, I.R. Klebanov and J.M. Maldacena, *Nucl. Phys.* **B 472** (1996) 231 [hep-th/9601057];
A. Hashimoto and I. Klebanov, *Nucl. Phys.* **55** *(Proc. Suppl.)* (1997) 118 [hep-th/9611214].

[8] J.L.F. Barbon, *Phys. Lett.* **B 382** (1996) 60 [hep-th/960198].

[9] M.R. Garousi and R.C. Myers, *Nucl. Phys.* **B 475** (1996) 193 [hep-th/9603194].

[10] R.G. Leigh, *Mod. Phys. Lett.* **A 428** (1989) 2767.

JHEP12(1998)008

[11] M.R. Douglas, *Branes within branes,* hep-th/9512077.

[12] S. Gukov, I.R. Klebanov and A.M. Polyakov, *Dynamics of (n,1) strings,* *Phys. Lett.* **B 423** (1998) 64 [hep-th/9711112].

[13] J.C. Breckenridge, G. Michaud and R.C. Myers, *Phys. Rev.* **D 55** (1997) 6438 [hep-th/9611174].

[14] M.E. Peskin, *Introduction to string and superstring theory II,* in *From the planck scale to the weak scale: toward a theory of the universe,* Proceedings of TASI '86, H. Haber ed., World Scientific Publishing 1987;

V.A. Kostelecky, O. Lechtenfeld, W. Lerche, S. Samuel and S. Watamura, *Nucl. Phys.* **B 288** (1987) 173.

[15] H. Arfaei and M.M. Sheikh Jabbari, *Mixed boundary conditions and brane, string bound states,* *Nucl. Phys.* **B 526** (1998) 278 [hep-th/9709054].

[16] C.G. Callan, C. Lovelace, C.R. Nappi and S.A. Yost, *Nucl. Phys.* **B 308** (1988) 221;

P. Di Vecchia, M. Frau, I. Pesando, S. Scituo, A. Lerda and R. Russo, *Nucl. Phys.* **B 507** (1997) 259 [hep-th/9707068].

[17] J.H. Schwarz, *Phys. Rep.* **89** (1982) 223.

[18] D. Friedan, *Notes on string theory and two dimensional conformal field theory,* in *Unified string theories,* M.B. Green and D.J. Gross eds., World Scientific Publishing 1986.

[19] D. Friedan, S. Shenker and E. Martinec, *Phys. Lett.* **B 160** (1985) 55; *Nucl. Phys.* **B 271** (1986) 93.

[20] M. Veltman, in *Methods in field theory,* Les Houches 1975 R. Balian and J. Zinn-Justin eds., North Holland 1976.

[21] E. Bergshoeff, H.J. Boonstra and T. Ortín, *Phys. Rev.* **D 53** (1996) 7206 [hep-th/9508091].

[22] M.R. Garousi and R.C. Myers, *World-volume interactions of D-branes,* hep-th/9809100.

[23] A. Hashimoto and I.R. Klebanov, *Phys. Lett.* **B 381** (1996) 437 [hep-th/9604065].

[24] A.A. Tseytlin, *Nucl. Phys.* **B 501** (1997) 41 [hep-th/9701125].

[25] A. Hashimoto and W. Taylor IV, *Fluctuation spectra of tilted and intersecting D-branes from the Born-Infeld action,* *Nucl. Phys.* **B 503** (1997) 193 [hep-th/9703217].